PREDICTING THE POPULATION VALUE OF THE CANONICAL REDUNDANCY STATISTIC

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The basis for canonical correlation analysis was developed by Hotelling (1935, 1936). He defined "the most predictable criterion" as the linear combination of criterion variables that is predicted by a linear combination of predictor variables so that the two linear combinations have the highest possible correlation. When the influence of the first two linear combinations is partialled out, the process is repeated on the residuals, thus obtaining a sequence of pairs of variates with maximum correlations between them. These were denoted by Hotelling as canonical variates and canonical correlations, respectively.

Many authors subsequently noted that determination of the "most predictable criterion" is not always the appropriate goal of educational or psychological research. However, it was immediately appreciated that the technique developed by Hotelling provides a mechanism to study the number and nature of mutually independent relations between two sets of variables.

Darlington, Weinberg and Walberg (1973) described the manner in which canonical correlation analysis assists in researching such relationships. First, it determines the minimum number of traits needed to account for the important linear relationships between two batteries. For example, a researcher might hypothesize that there are r traits that describe the important relationships between a set of attitude variables and a set of performance variables. After performing a canonical correlation analysis, the number of significant relationships may be determined by testing whether the canonical correlation coefficient is greater than zero. Second, the standardized weights and factor structures assist in describing the nature of these traits.

An outline of the eigenanalysis procedure used in canonical correlation is presented to facilitate the following discussions.

Let $\underline{R}_{xx}$ be the matrix of correlations between the p predictor variables;

$\underline{R}_{yy}$ be the matrix of correlations between the q criterion variables;

$\underline{R}_{xy}$ be the matrix of correlations between the predictor and criterion variables.

The canonical correlation solution is obtained from eigenanalysis of the nonsymmetric matrix formed by the product $\underline{R}_{xx}^{-1}\underline{R}_{xy}\underline{R}_{yy}^{-1}\underline{R}_{yx}$.

Let $\underline{\Lambda}$ be the resulting diagonal matrix of eigenvalues with the ith diagonal element denoted λ_i^2;

$\underline{A}$ be the resulting matrix of eigenvectors for the predictor variables with the ith column denoted $\underline{a}_i$;

$\underline{B}$ be the resulting matrix of eigenvectors for the criterion variables resulting from eigenanalysis of $\underline{R}_{yy}^{-1}\underline{R}_{yx}\underline{R}_{xx}^{-1}\underline{R}_{xy}$ with the ith column denoted b_i.

Then $v_{xi} = \underline{a}_i'\underline{x} = a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{ip}x_p$ is the ith canonical variate for the predictor variables, i=1, ..., s, where 2=min(p,q);

$v_{yj} = \underline{b}_j'\underline{y} = b_{j1}y_1 + b_{j2}y_2 + \cdots + b_{jq}y_q$ is the jth canonical variate for the criterion variables, j=1, ..., 2.

The eigenvalues are the squared canonical correlation coefficients between each successive pair of canonical variates. The eigenvector matrices, $\underline{A}$ and $\underline{B}$, contain the standardized weights for the predictor and criterion variables, respectively, and are used to form the canonical variates. If eigenanalysis is performed using variance-covariance matrices in place of correlation matrices the eigenvalue matrix must be postmultiplied by the diagonal matrix of standard deviations of the appropriate set of variables, predictor or criterion, to obtain the standardized weights.

As with any statistical technique, researchers must have a mechanism to judge the statistical and practical significance of the results of canonical correlation analysis. The standardized weights, $\underline{A}$ and $\underline{B}$, and the factor structure correlations between each canonical variate and the original variables, $\underline{R}_{xx}\underline{A}$ and $\underline{R}_{yy}\underline{B}$, may be examined to interpret the relationship between canonical variates and the original measures. The squared correlation, λ^2, between each pair of canonical variates may be interpreted as the amount of variance shared by the two linear combinations of the predictor and criterion variables. However, these statistics fail to provide information regarding the amount of shared variation between the variables in the two batteries. Stewart and Love (1968) and Miller (1969), with the consultation of Paul Lohnes, developed a measure to permit this type of interpretation. Their statistic, denoted the bimultivariate redundancy index, or the canonical redundancy index in the special case of canonical correlation, has intuitive appeal. The canonical redundancy statistic is defined as the sum of successive products between the proportion of variance that the canonical variates of either battery explain in the canonical variates of the other (accounted for by the squared canonical correlations), and the proportion of variance absorbed by the canonical variates from their respective batteries (accounted for by the squared correlations between the variates and original variables). Using the above notation, the total canonical redundancy statistic for the predictor variables, given the criterion variables, is

$$Rd_x = \frac{1}{p} \Sigma_j^s \, (\underline{R}_{xx}\underline{a}_j)' \, (\underline{R}_{xx}\underline{a}_j) \, \lambda_j^2;$$

the total canonical redundancy statistic for the criterion variables, given the predictor variables, is similarly

$$Rd_y = \frac{1}{q} \Sigma_j^s (\underline{R}_{yy}\underline{b}_j)' (\underline{R}_{yy}\underline{b}_j) \lambda_j^2;$$

where $s = \min(p,q)$.

This sum determines the amount of redundancy in one battery of variables given the other. As such, it is directional and nonsymmetric and has a desirable range of zero to one.

Miller (1969) and Miller and Farr (1971) demonstrated the equivalence of the total redundancy measures based upon multiple regression of independently orthogonalized batteries, such as in principle component analysis, and the total redundancy measures based upon canonical correlation analysis, a simultaneous orthogonalization procedure. The two solutions differ, however, in the structural components of the batteries and, therefore, the redundancy measures for individual components or variates are not identical.

Gleason (1976) established the mathematical basis for a generalized version of the canonical redundancy statistic. One way of interpreting redundancy of one set of variables, given another set, is to reconstruct one set using only the information in the second set that is relevant to that in the first set. Gleason demonstrated that this approach leads to a mathematical expression that is equivalent to the canonical redundancy index as defined by Stewart and Love (1968), and thus provides the mathematical rigor for the definition of this measure.

The recent development of an index that describes the overlap or amount of redundancy between two batteries in canonical correlation analysis is extremely welcome. The need for such a measure is apparent, and researchers in education and psychology are generally eager to utilize measures that assist them in their studies. In the short period of time since the papers by Stewart and Love (1968) and Miller (1969) were published, the canonical redundancy statistic has been described and recommended in texts and articles by some of social sciences' leading authors. For examples, see Cooley and Lohnes (1971, p. 170-172; Tatsuoka (1973, p. 280-282); Cohen and Cohen (1975, p. 429-432); Timm (1975, p. 355-358) and Cooley and Lohnes (1976, p. 211-212). In addition, an entire session at the 1976 Annual Meeting of the American Education Research Association was devoted to applied research on the redundancy statistic.

It is almost a certainty that the redundancy statistic is positively biased. First, consider the dependence of this statistic on the squared canonical correlation coefficient. Second, the results by Miller (reported in Cooley and Lohnes, 1976) of a Monte Carlo analysis investigating the sampling distribution in the null case indicate bias of the median total redundancy statistic ranging from .06 to .09 for various combinations of numbers of predictor and criterion variables and sample sizes. No other data are available on the bias of this statistic. Thus the purpose of the present study was to investigate the empirical sampling distribution of the first squared canonical correlation coefficient and of the redundancy statistic using Monte Carlo methods; subsequently, an attempt was made to derive a formula to correct for bias in the total redundancy statistic.

The investigation entailed the systematic variation of the number of predictor and criterion variables, the sample size, the size of the intrabattery correlations and the size of the interbattery correlations since these are the six parameters that affect the magnitude of the canonical redundancy statistic.

Two levels of the number of variables were designated for each of the left and right sets: cases with five and nine variables for each set. Due to the symmetry of canonical correlation analysis, it was necessary to consider only the following three combinations:

$$\{(p,q)\} = \{(5,5), (9,5), (9,9)\}$$

Two sample sizes were selected for study, the case of a small sample and the case of a large sample. For small n, the value used was 5(p+q), a sample size frequently encountered in applied research. For the large sample, n equal to 20(p+q) was used to examine effects of a sample size frequently recommended. Thus the sample sizes were as follow:

$(p,q) = (5,5);\ n_s = 50,\ n_\ell = 200$
$(p,q) = (9,5);\ n_s = 70,\ n_\ell = 280$
$(p,q) = (9,9);\ n_s = 90,\ n_\ell = 360$

Two conditions were chosen for the average intercorrelations of each of the matrices $\underline{P}_{xx}$ and $\underline{P}_{yy}$, and three conditions were selected for $\underline{P}_{xy}$. In applied research, variables in the predictor set often have medium to high correlations. Similarly for the intercorrelations between criterion variables. However, the correlations between predictor and criterion variables are quite often lower. In an attempt to reflect conditions often encountered in actual research, the off-diagonal elements of $\underline{P}_{xx}$ and $\underline{P}_{yy}$ were set to .30 and .60 to reflect medium and high correlations respectively. All of the elements of $\underline{P}_{xy}$ were set to .00, .20 and .40 to reflect the null case, low correlations and medium correlations, respectively. The inclusion of the null case for no relationship between the two sets of variables was important in this study to provide baseline information against which to compare bias in the non-null cases. The fact that $\underline{P}_{xy} = \underline{P}_{yx}'$ was utilized to form the ((p+q)x(p+q)) supermatrix

$$\underline{P} = \begin{pmatrix} \underline{P}_{xx} & \underline{P}_{xy} \\ \underline{P}_{yx} & \underline{P}_{yy} \end{pmatrix}.$$

The above conditions lead to the definition of 36 population matrices. (Three combinations of numbers of predictor and criterion variables, 2 levels of $\underline{P}_{xx}$, 2 levels of $\underline{P}_{yy}$, and 3 levels of $\underline{P}_{xy}$.) Calculation of parameters and statistics based upon 2 sample sizes increases the number of specific situations under investigation to 72. The process used to define population conditions and generate sample matrices for the Monte Carlo analysis is presented in schematic form in Figure 1.

The results of the Monte Carlo study show that considerable positive bias is obtained when a sample redundancy statistic is used to estimate the population value. In general, the amount of bias for the redundancy statistic defined on one battery tends to decrease as the number of variables increase in the second battery. Bias appears to be unaffected by the number of variables when this number is equal for both batteries. Bias of both the redundancy statistic and the largest squared canonical correlation coefficient is consistent for all levels of intra-battery correlation but decreases as interbattery correlations increase, indicating less bias in the non-null cases. The most dramatic parameter affective bias is, as might be expected, sample size. Bias increases approximately fourfold as the sample size increases by the same amount. It is not known whether this relationship is linear since a sufficient number of sample sizes were not considered in the present study.

It is useful in the case of a biased estimate to employ a formula that "corrects" the estimate and provides a value that is closer to the population parameter. The present study utilized two approaches to attempt to estimate the population value of the total redundancy statistic, given information about the sample. One approach applied two standard shrinkage formulae to the sample value; the other regressed the population value on sample information. The results of the regression analysis are presented first.

The intrabattery and interbattery correlations were recoded for purpose of the regression analysis. Two regression equations were calculated, the first using the following variables and values:

p: 5 or 9
q: 5 or 9
R_{xx}: 1 if $R_{xx} = .30$, 2 if $R_{xx} = .60$
R_{yy}: 1 if $R_{yy} = .30$, 2 if $R_{yy} = .60$
R_{xy}: 0 if $R_{xy} = .00$, 1 if $R_{xy} = .20$, 2 if $R_{xy} = .40$
n: 5(p+q) or 20(p+q).

The values for intrabattery and interbattery correlations were recoded as above to attempt development of a regression equation that would be more generalizable. Indeed, the matrix randomly generated from the population conditions did not have intra- and interbattery correlations precisely equal to the population values. The computer algorithm routine used (Montanelli, 1971) generates sample correlation matrices that would result from sampling random normal variables having the required population correlation structure. Thus the actual matrices used as the population had correlations that, upon repeated sampling, have expected values equal to the population values of .30, .60 for intrabattery correlations and .00, .20 and .40 for interbattery correlations, respectively.

The second regression analysis included the mean sample value of the redundancy statistic in addition to the predictor variables listed above. The 72 values of Rd_x and Rd_y were combined to give a sample of N=144 for the analysis. The regression analysis resulted in multiple correlation coefficients of R=.9925 and R=.7904 for the regression equations computed with and without the mean sample value, respectively. Beta weights were tested for significant contribution to the regression equation, and only those variables with corresponding beta weights significant at $\alpha \leq .01$ were retained. The significant beta weights were then rounded to three decimal places for all but one variable and a predicted value for each of the 144 sets of observations was computed. This value was then correlated with the population value to obtain an adjusted Multiple R.

The results of the regression analysis are contained in Table 1. The use of the reduced set of weights does not affect the Multiple R significantly. It is perhaps unfortunate that the population value is not better predicted without the sample mean. This may be an artifact of the restricted upper range of these statistics, although population values were not that large in the present study, the largest being 0.502. Another reason for the poor prediction without the sample mean is that the population value does not vary with the sample size while it is apparent from detailed Monte Carlo results that the degree of bias is greatly affected by this variable. Consequently, it is not surprising that sample size was not significant in the regression analysis omitting the sample mean. However, with the sample mean included in the set of predictors, the sample size is a significant predictor, as might be expected.

The results of the Monte Carlo Analyses indicate that the behavior of the bias of total redundancy statistic is quite similar to that of the squared canonical correlation coefficient. In addition, Miller (1975b, 1976) demonstrated that the redundancy statistics are approximated by an F distribution with modified degrees of freedom in the null case. It was therefore decided to apply the Wherry and the Olkin-Pratt (Kendall and Stuart, 1967) shrinkage formulae for the squared multiple correlation coefficient in the hopes that the population values of the redundancy statistics may be similarly estimated*. The significant increase in the Multiple R achieved when the sample mean value of the canonical redundancy statistic was included in the regression analysis was a further indication that investigation of these formulae, which utilize the sample value, might be worthwhile.

The formulae used were the following:
Let N be the sample size,
q be the number of variables in the criterion set,
Rd be the total redundancy statistic for the predictor set given the criterion set,

* The author would like to express her appreciation to John Pohlmann, Southern Illinois University, for his suggestion to examine the efficacy of these formulae.

Then

$$\text{Wherry correction} = 1 - \frac{N-1}{N-q-1}(1-Rd);$$

$$\text{Olkin-Pratt correction} = 1 - \frac{N-3}{N-q-1}(1-Rd) - \left(\frac{N-3}{N-q-1}\right)\left(\frac{2}{N-q+1}\right)(1-Rd)^2.$$

In the case of the total redundancy for the criterion set given the predictor set, the roles of p and q were exchanged to be consistent with the above.

Each formula was applied to the mean sample values of the total redundancy statistic and the difference between the population value and the shrunken estimate was calculated. In addition the mean absolute value of the difference was computed over all 144 values of Rd. Table 2 contains the results of this analysis.

Both formulae provide excellent approximations to the population value. The average absolute values of the difference between the population and the corrected value were 0.003347 for the Wherry formula and 0.001955 for the Olkin-Pratt formula. Although the Olkin-Pratt formula is better when considering only the residuals, Table 2 illustrates that this formula results in more overestimates of the population values. This is evidenced by the larger number (82) of negative differences as compared to only 12 overestimates using the Wherry formula. With the Wherry formula, only 3 of the 144 estimates vary from the population parameter by a value greater than 0.01, and these are all underestimating the parameter. Thus, while both formulae provide excellent corrections for the total redundancy statistic, the Wherry is recommended due to its tendency to provide a conservative estimate.

It would seem that the results of the Monte Carlo analysis do not justify the recommendation of the canonical redundancy statistic as an alternative to the squared canonical correlations on the basis of less bias. The redundancy statistic, in general, appears to exhibit a degree of bias quite similar to that of the squared canonical correlation. However, the bias is easily corrected by the Wherry or Olkin-Pratt formulae to estimate the true population value. What is perhaps more important are the interpretive characteristics of the canonical redundancy statistic as compared to those of the canonical correlation coefficient. If interest truly lies in the relationship between groups of variables as opposed to the relationship between linear combinations of variables, then the redundancy statistic provides a more realistic and meaningful measure for the conscientious education researcher.

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FIGURE 1: SCHEMATIC REPRESENTATION OF DESIGN OF MONTE CARLO ANALYSIS

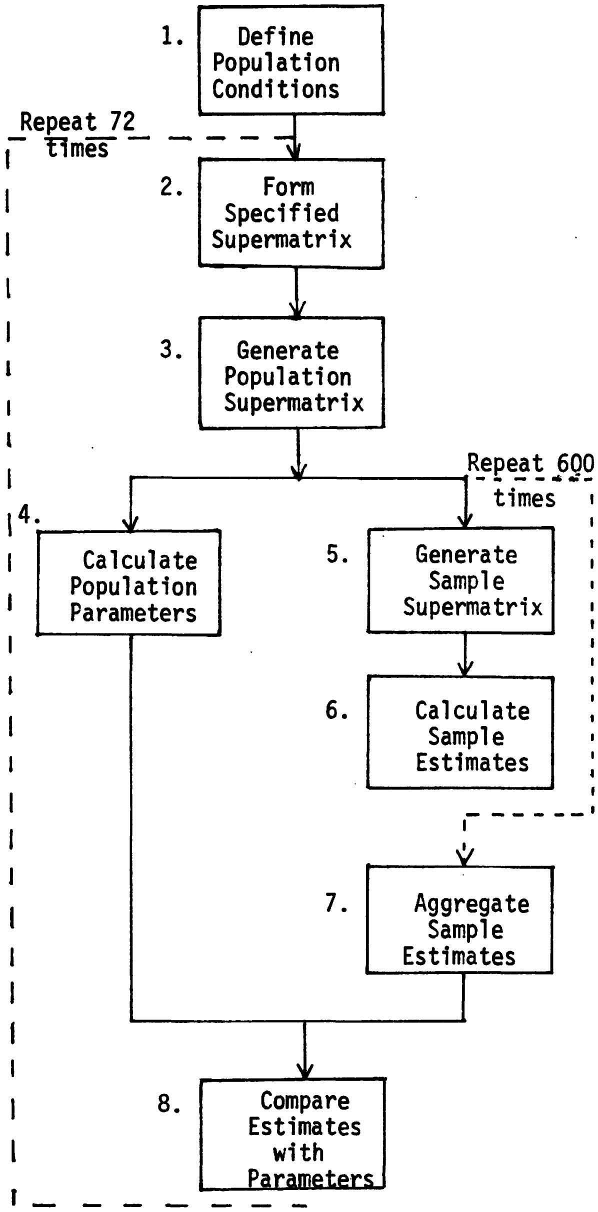

TABLE 1: REGRESSION OF TOTAL REDUNDANCY POPULATION VALUE ON SAMPLE INFORMATION

NOT INCLUDING THE SAMPLE MEAN AS A PREDICTOR:

VARIABLE	ORIGINAL β WEIGHT	F
p	0.0102	9.15
q	-0.0007	0.04
R_{xx}	-0.0496	21.01
R_{yy}	-0.0372	11.80
R_{xy}	0.0897	183.36
n	-0.0000	0.00
Constant	0.1775	

Multiple R = .7904

Adjusted Multiple R = .7903

INCLUDING THE MEAN SAMPLE VALUE AS A PREDICTOR:

VARIABLE	ORIGINAL β WEIGHT	F
p	-0.0025	12.18
q	-0.0033	23.86
R_{xx}	-0.0032	1.95
R_{yy}	-0.0024	1.11
R_{xy}	0.0056	7.84
n	0.0003	541.45
$\overline{X}$	0.9984	3261.739
Constant	-0.0484	

Multiple R = .9925

Adjusted Multiple R = .9911

TABLE 2

WHERRY AND OLKIN/PRATT CORRECTION FORMULAE APPLIED TO THE TOTAL REDUNDANCY STATISTIC

p	q	n	POPULATION VALUE	POPULATION-WHERRY	POPULATION-OLKIN/PRATT	p	q	n	POPULATION VALUE	POPULATION-WHERRY	POPULATION-OLKIN/PRATT
5	5	50	0.143963	0.003304	-0.004118	5	5	50	0.158364	0.003330	-0.004422
5	5	50	0.140473	-0.000261	-0.007684	5	5	50	0.159670	0.000585	-0.007257
5	5	50	0.409582	0.008356	-0.002657	5	5	50	0.368618	0.008368	-0.002417
5	5	50	0.143963	0.007461	0.000138	5	5	50	0.169623	0.005078	-0.002884
5	5	50	0.119790	0.004827	-0.001965	5	5	50	0.153155	0.010905	0.003447
5	5	50	0.251462	0.006330	-0.003142	5	5	50	0.339137	0.006596	-0.003964
5	5	50	0.148249	0.005880	-0.001581	5	5	50	0.158365	0.005209	-0.002500
5	5	50	0.097611	0.007781	0.001653	5	5	50	0.131802	0.007002	-0.000037
5	5	50	0.257855	0.004125	-0.005480	5	5	50	0.217230	0.004450	-0.004474
5	5	50	0.148248	0.000872	-0.006706	5	5	50	0.169623	0.005690	-0.002259
5	5	50	0.088609	0.003768	-0.002222	5	5	50	0.125883	0.006079	-0.000835
5	5	50	0.140648	0.007815	0.000580	5	5	50	0.188901	0.006999	-0.001329

TABLE 2 (continued)

WHERRY AND OLKIN/PRATT CORRECTION FORMULAE APPLIED TO THE TOTAL REDUNDANCY STATISTIC

p	q	n	POPULATION VALUE	POPULATION-WHERRY	POPULATION-OLKIN/PRATT
5	5	200	0.143963	0.001337	-0.000041
5	5	200	0.140473	0.001132	-0.000223
5	5	200	0.409582	0.002325	-0.000172
5	5	200	0.143963	0.002531	0.001161
5	5	200	0.119790	0.001502	0.000297
5	5	200	0.251462	0.001453	-0.000545
5	5	200	0.148249	0.002995	0.001599
5	5	200	0.097611	0.001712	0.000675
5	5	200	0.257855	0.001999	-0.000027
5	5	200	0.148248	0.001202	-0.000206
5	5	200	0.088609	0.002333	0.001371
5	5	200	0.140648	0.001889	0.000538
5	5	200	0.158364	0.000991	-0.000485
5	5	200	0.159670	0.001988	0.000509
5	5	200	0.368618	0.003466	0.001054
5	5	200	0.169623	0.001568	0.000023
5	5	200	0.153155	0.001228	-0.000212
5	5	200	0.339137	0.001201	-0.001137
5	5	200	0.158365	0.000399	-0.001082
5	5	200	0.131802	0.001810	0.000521
5	5	200	0.217230	0.003660	0.001847
5	5	200	0.169623	0.000722	-0.000829
5	5	200	0.125883	0.000900	-0.000354
5	5	200	0.188901	0.001900	0.000238
9	5	70	0.126378	0.005372	0.000981
9	5	70	0.177453	0.006378	0.001104
9	5	70	0.404501	0.004114	-0.003453
9	5	70	0.126366	0.006023	0.001645
9	5	70	0.140718	-0.000907	-0.005678
9	5	70	0.291738	0.005802	-0.000980
9	5	70	0.135215	0.003134	-0.001464
9	5	70	0.140720	-0.000963	-0.005735
9	5	70	0.358482	0.011438	0.004148
9	5	70	0.135185	0.006471	0.001935
9	5	70	0.117356	0.004498	0.000263
9	5	70	0.216770	0.007672	0.001819
9	5	70	0.185906	0.005070	-0.000397
9	5	70	0.235254	0.006339	0.000183
9	5	70	0.502448	0.004181	-0.003505
9	5	70	0.178609	0.002624	-0.002767
9	5	70	0.195848	0.001877	-0.003791
9	5	70	0.468800	0.007413	-0.000298
9	5	70	0.185910	0.006682	0.001240
9	5	70	0.185180	0.002131	-0.003371
9	5	70	0.311414	0.009326	0.002363
9	5	70	0.178601	0.006940	0.001619
9	5	70	0.155714	0.001594	-0.003436
9	5	70	0.255294	0.007934	0.001547
9	5	280	0.126378	0.000311	-0.000557
9	5	280	0.177453	0.001906	0.000798
9	5	280	0.404501	0.001089	-0.000673
9	5	280	0.126366	0.001165	0.000301
9	5	280	0.140718	0.001171	0.000234
9	5	280	0.291738	0.001049	-0.000481
9	5	280	0.135215	0.000048	-0.000867
9	5	280	0.140720	0.001432	0.000497
9	5	280	0.358482	-0.000552	-0.002245
9	5	280	0.135185	0.002048	0.001143
9	5	280	0.117356	-0.000014	-0.000837
9	5	280	0.216770	0.001037	-0.000240
9	5	280	0.185906	0.001482	0.000335
9	5	280	0.235254	0.001258	-0.000087
9	5	280	0.502448	0.001689	-0.000129
9	5	280	0.178609	0.000362	-0.000758
9	5	280	0.195848	0.002733	0.001549
9	5	280	0.468800	0.000971	-0.000843
9	5	280	0.185910	0.000338	-0.000814
9	5	280	0.185180	0.002177	0.001037
9	5	280	0.311414	0.000169	-0.001417
9	5	280	0.178601	0.002573	0.001463
9	5	280	0.155714	0.000544	-0.000469
9	5	280	0.255294	0.000875	-0.000543
9	9	90	0.146454	0.009138	0.005702
9	9	90	0.211141	0.004578	0.000240
9	9	90	0.464035	0.007861	0.001979
9	9	90	0.146455	0.007755	0.004299
9	9	90	0.160521	0.002294	-0.001437
9	9	90	0.337630	0.003364	-0.002098
9	9	90	0.157883	0.013028	0.009483
9	9	90	0.181501	0.000256	-0.003776
9	9	90	0.452014	0.004943	-0.000930
9	9	90	0.157884	0.009940	0.006352
9	9	90	0.142738	0.002298	-0.001183
9	9	90	0.262638	0.000827	-0.004083
9	9	90	0.125934	0.007711	0.004559
9	9	90	0.196522	-0.000012	-0.004232
9	9	90	0.379337	0.006317	0.000653
9	9	90	0.118614	0.004445	0.001356
9	9	90	0.153542	0.002280	-0.001355
9	9	90	0.453077	0.001585	-0.004293
9	9	90	0.130175	0.008485	0.005281
9	9	90	0.150499	-0.000666	-0.004299
9	9	90	0.325202	0.000843	-0.004557
9	9	90	0.122898	0.006177	0.003048
9	9	90	0.119296	0.001248	-0.001901
9	9	90	0.248147	0.002326	-0.002432
9	9	360	0.146454	0.003555	0.002827
9	9	360	0.211141	0.002434	0.001475
9	9	360	0.464035	-0.000382	-0.001786
9	9	360	0.146455	0.003094	0.002364
9	9	360	0.160521	0.000214	-0.000581
9	9	360	0.337630	0.001363	0.000092
9	9	360	0.157883	0.005156	0.004390
9	9	360	0.181501	-0.000246	-0.001116
9	9	360	0.452014	0.000755	-0.000644
9	9	360	0.157884	0.005443	0.004678
9	9	360	0.142738	0.002636	0.001919
9	9	360	0.262638	-0.000884	-0.001999
9	9	360	0.125934	0.002309	0.001657
9	9	360	0.196522	0.000495	-0.000423
9	9	360	0.379337	0.001064	-0.000270
9	9	360	0.118614	0.001321	0.000695
9	9	360	0.153542	0.001024	0.000259
9	9	360	0.453077	0.001321	-0.000078
9	9	360	0.130175	0.003793	0.003130
9	9	360	0.150499	-0.000119	-0.000876
9	9	360	0.325202	-0.000193	-0.001445
9	9	360	0.122898	0.004237	0.003605
9	9	360	0.119296	0.005171	0.004559
9	9	360	0.248147	0.001237	0.000166